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CASM: A UNIFIED STATE PARAMETER MODEL FOR CLAY AND SAND

H. S. YU*

Department of Civil Engineering and Surveying, The University of Newcastle, N.S.W. 2308, Australia

SUMMARY

The purpose of this paper is to present a simple, unified critical state constitutive model for both clay and sand. The model, called CASM (Clay And Sand Model), is formulated in terms of the state parameter that is defined as the vertical distance between current state (v, p') and the critical state line in v–$ln\,p'$ space. The paper first shows that the standard Cam-clay models (i.e. the original and modified Cam-clay models) can be reformulated in terms of the state parameter. Although the standard Cam-clay models prove to be successful in modelling normally consolidated clays, it is well known that they cannot predict many important features of the behavior of sands and overconsolidated clays. By adopting a general stress ratio-state parameter relation to describe the state boundary surface of soils, it is shown that a simple, unified constitutive model (CASM) can be developed for both clay and sand. It is also demonstrated that the standard Cam-clay yield surfaces can be either recovered or approximated as special cases of the yield locus assumed in CASM.

The main feature of the proposed model is that a single set of yield and plastic potential functions has been used to model the behaviour of clay and sand under both drained and undrained loading conditions. In addition, it is shown that the behaviour of overconsolidated clays can also be satisfactorily modelled. Simplicity is a major advantage of the present state parameter model, as only two new material constants need to be introduced when compared with the standard Cam-clay models. © 1998 John Wiley & Sons, Ltd.

Key words: state parameter; stress–state relation; constitutive modelling; plasticity; critical state; sand and clay

INTRODUCTION

The critical state theory was first used to develop plasticity models for soils over 30 years ago (see, for example, References 1–5). Since then, elastic–plastic models based on the critical state concept have been successfully used to describe many important features of soil behaviour. The original Cam-clay model was developed by Roscoe and Schofield[3] and the detail of this model can also be found in Reference.[4] Later, Roscoe and Burland[5] proposed a modified Cam-clay model and its generalization to three dimensions.

To achieve a better agreement between predicted and observed soil behaviour, a large number of modifications have been proposed to the standard Cam-clay models over the last two decades. A bried review on some of the most important modifications may be found in Reference.[6]

*Correspondence to: Department of Civil Engineering and Surveying, The University of Newcastle, N.S.W. 2308, Australia, E-mail: cehsy@cc.newcastle.edu.au

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Examples of these modifications include research on the following topics: (a) yield surface for heavily overconsolidated clays;[7,8] (b) critical state modelling of sand behaviour;[9,10] (c) anisotropic yield surfaces for one-dimensionally consolidated soils;[11,12] (d) inclusion of plastic deformation within the main yield surface for soils subject to cyclic loading;[13–16] (e) 3D critical state formulations;[5,17] and (f) modelling of rate-dependent behaviour of clays.[18,19]

Despite much of the success in modifying the standard Cam-clay models over the last 20 years, the following problems still remain:

(1) The yield surfaces adopted in many critical state models significantly overestimate failure stresses on the 'dry side'. To overcome this limitation, the Hvorslev surface is often used as the yield function in this region.[7] The problem with this treatment is that there will be two separate yield surfaces for hardening and softening, and this discontinuity in the yield surface will cause significant numerical difficulties.[16] This is probably why the Hvorslev surface is seldom implemented in geotechnical computer softwares.[6]
(2) Many critical state soil models assumed an associated flow rule and therefore were unable to predict an important feature of behaviour that is commonly observed in undrained tests on loose sand and normally consolidated undisturbed clays, and that is a peak in the deviatoric stress before the critical state is approached.[10,20–22] Furthermore, as shown by Vermeer,[23] no bifurcation is possible in the hardening regime if an associated flow rule is used, and this contradicts with experimental observations.[24]
(3) The critical state concept has been much less successful for modelling granular materials.[4,9,25–27] The main problem lies in the fact that existing Cam-clay models fail to predict observed softening and dilatancy of dense sands and undrained response of very loose sands. The lack of success in developing a critical state model for sand is also due to the experimental difficulties in obtaining critical state and normal consolidation lines.[8] Until very recently, little data for the critical state and normally consolidation lines of sands was available. In addition, experimental data for sands seems to support a different picture of yield surfaces from that seen for clays.[28–30]

The above observation are confirmed by Gens and Potts[6] in their state of the art review on the application of critical state models in computational geomechanics, where they noted:

(1) The materials modelled by critical state models appear to be mostly limited to saturated clays and silts, and stiff overconsolidated clays do not appear to be generally modelled with critical state formulations. This fact is probably related to the poor performance of critical state models for soils on the 'dry side'.
(2) Granular materials are rarely modelled by critical state models. In spite of the fact that a number of 'double hardening' sand models[9,31,32] have been available for many years they do not appear to have been widely used in numerical analyses. This is partially due to the fact that two separate yield surfaces are used in these models for modelling hardening (or consolidation yielding) and softening (or shear yielding), which tends to cause significant numerical difficulties. Another reason may be that a large number of constants (some of these constants have no clear physical meaning) need to be determined before these sand models can be applied.

As noted by Scott,[33] there is an undesirable trend in recent years in the area of constitutive modelling of soils to increase the number of constitutive constants (some models now use as many as 40 material constants). Apart from the drawback that many of the material constants offer no

clear physical meaning, there is a difficulty that representation of a specific phenomenon is governed by a number of constants. Hence, the values of certain constants cannot be determined independently of the others. Even if some of these models are considered to be very successful in modelling soil behaviour, the large number of material constants required will make them very hard to apply to practical problems. In contrast, the philosophy adopted in this paper in choosing the structure of the model, is that simplicity should be paramount and that the material constants required by the model should be related to easily measurable, possibly, conventional constants. In the spirit of the hierarchical approach of Desai *et al.*,[34] the present model is formulated in such a way that the standard Cam-clay models can be recovered (or approximated) simply by choosing certain values of the material constants. Contrary to many existing critical state models[10,35] that use distinctively different yield functions and plastic potentials for clay and sand, a single yield function and plastic potential are used in this paper for both clay and sand. As will be shown, this can be achieved by using a general stress–state relation to derive a unified state boundary surface.

The main objective of this paper is to develop a unified critical state theory for both clay and sand, and the proposed model aims to link the plastic behaviour of the soil during shear and consolidation in a rational manner. The development of the present critical state theory is motivated by extensive experimental work conducted recently by Been and his co-workers,[20,36–38] Sladen *et al.*[21] and Coop and Lee[39] on various granular materials. The number of material constants required is two more than that required by the standard Cam-clay models. All material constants have clear physical meanings and they are also relatively simple to determine in routine laboratory or field tests.

STATE PARAMETER CONCEPT

The state parameter is defined by Been and Jefferies[20] as the difference between specific volume (or void ratio) and the specific volume (or void ratio) at the critical state at the same mean effective stress, see Figure 1. This parameter was first used by Wroth and Bassett[25] in the development of a stress–strain relation for sand.

The experimental research on clay and dense sands by Roscoe and Poorooshasb,[40] Cole[41] and Stroud[42] suggests that any two samples of a soil will behave in a similar manner regardless of their stress–strain history provided the state parameter is the same for each sample. Recent work on dense and loose sands by Been and Jefferies,[22] Sladen *et al.*[21] and Sladen and Oswell[43] confirms this observation and suggests that the state can be confidently used to describe much of the behaviour of granular materials over a wide range of stresses and densities. This is because the state parameter does not eliminate the influence of either density or confining pressure on the behaviour of sands, and rather it properly places emphasis on the fact that it is a combination of these parameters that is relevant to the description of granular materials. It has also been demonstrated by Been and his co-workers that many commonly used sand properties, such as angles of friction and dilation, normalize quite well to the state parameter, and this is the utility of the concept to practising engineers.

It is well established that most sands behave rather like a clay at high OCR–on shearing they tend to dilate, although some sand deposits may be sufficiently loose to compress on shearing like a clay at low OCR. As a direct relationship between the OCR and the state parameter can be generally established,[44] it is expected that the state parameter for sand will play a similar role to the overconsolidation ratio (OCR) for clay. In clays, the OCR has been used as the main quantity to describe the character of clay response under given loading conditions. By comparison, the role

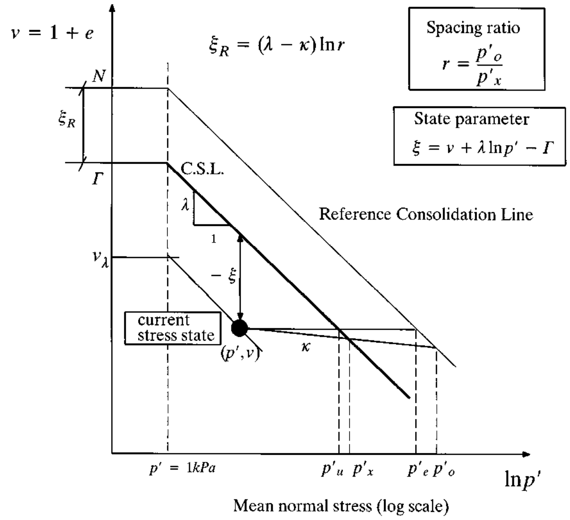

Figure 1. Definitions of state parameter, critical state constants and reference state parameter

of the OCR in sands is less clear, mainly because the preconsolidation pressure can only be determined accurately at high pressures. As an alternative, Stroud[42] shows that a similar type of information is available if the density and the current pressure are known (i.e. the state parameter). Because the state parameter may be determined easily for both clay and sand, it may be regarded as a better quantity than OCR for describing soil response under various loading conditions.

CAM-CLAY MODELS IN TERMS OF STRESS-STATE RELATIONS

Like many critical state models, we shall start our development for the case of a triaxial test, that is, when two effective principal stresses are equal and the directions of principal stresses are fixed with respect to the material element. Assuming compressive stresses and strains as positive, the two stress variables normally used in the critical state soil mechanics are:

$$p' = \frac{\sigma_1 + 2\sigma_3}{3}, \qquad q = \sigma_1 - \sigma_3 \tag{1}$$

where σ_1, σ_3 denote effective vertical and radial stresses respectively. The corresponding strains are:

$$\varepsilon_p = \varepsilon_1 + 2\varepsilon_3, \qquad \varepsilon_q = \tfrac{2}{3}(\varepsilon_1 - \varepsilon_3) \tag{2}$$

The original Cam-clay model

In the standard Cam-clay models, it is assumed that the critical state is fully defined by

$$q = Mp' \tag{3}$$

$$v = \Gamma - \lambda \ln p' \tag{4}$$

where q, p' denote the deviatoric and mean effective stresses; λ, κ and Γ are well-known critical state constants that are defined in Figure 1; M is the slope of the critical state line in $p' - q$ space as discussed before, the material behaviour prior to the achievement of the critical state is assumed to be controlled by the state parameter which is defined by

$$\xi = v + \lambda \ln p' - \Gamma \tag{5}$$

where $v = (1 + e)$ is known as specific volume and e is void ratio. It is noted that the state parameter ξ is zero at the critical state, positive on the 'wet' side and negative on the 'dry' side.

The state boundary surface of the original Cam-clay model is presented by Schofield and Wroth[4] as follows:

$$\frac{q}{Mp'} = \frac{\Gamma + \lambda - \kappa - v - \lambda \ln p'}{\lambda - \kappa} \tag{6}$$

In the original Cam-clay model, equation (6) is used as a yield function. In addition, the consistency requirement that the differential of the yield function is zero is used to define a hardening law.

By using equation (5), the state boundary surface (6) may be expressed as a relationship between stress ratio and the state parameter, namely:

$$\frac{\eta}{M} = 1 - \frac{\xi}{\xi_{\mathrm{R}}} \tag{7}$$

where $\eta = q/p'$ is known as stress ratio, and ξ_{R} is a positive reference state parameter which denotes the vertical distance between the CSL and a reference consolidation line. As shown in Figure 1, the reference consolidation line is assumed to be parallel to CSL. For clays, the isotropic consolidation line, NCL, should be used as the reference consolidation line. In the original Cam-clay model, the reference state parameter is assumed to be $\xi_{\mathrm{R}} = (\lambda - \kappa) \ln r = (\lambda - \kappa) \ln e = \lambda - \kappa$, where r is known as the spacing ratio. For some sands, information about the NCL may not be accurately measured and in this case the reference state parameter may be chosen for a state beyond which the soil is unlikely to reach in practice. The stress–state relation (7) implies that when a soil is yielding, the stress ratio η increases linearly with a decrease in the state parameter, see Figure 2.

The modified Cam-clay model

The state boundary surface of the modified Cam-clay model was presented by Roscoe and Burland[5] and can be represented as follows:

$$\left(\frac{q}{Mp'}\right)^2 = \exp\left(\frac{N - v - \lambda \ln p'}{\lambda - \kappa}\right) - 1 \tag{8}$$

Equation (8) is used as a yield function in the modified Cam-clay model. Using equation (5), the state boundary surface (8) can be expressed as a modified stress–state relation, namely

$$\left(\frac{\eta}{M}\right)^2 = 2^{(1 - (\xi/\xi_{\mathrm{R}}))} - 1 \tag{9}$$

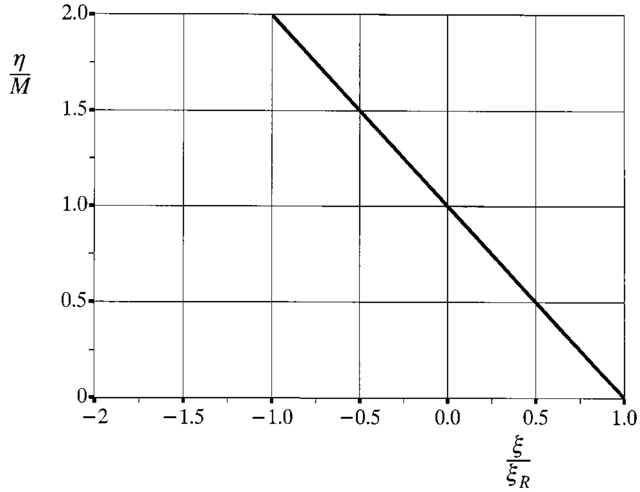

Figure 2. Stress–state relation for the original Cam-clay model with $\xi_R = (\lambda - \kappa)\ln e = \lambda - \kappa$

In the modified Cam-clay model, $\xi_R = (\lambda - \kappa)\ln r = (\lambda - \kappa)\ln 2$. Unlike the linear stress–state relation suggested by the original Cam-clay model, the modified stress–state relation (9) is non-linear, see Figure 3.

A GENERAL STRESS–STATE RELATION FOR BOTH CLAY AND SAND

A detailed study of the experimental state boundary surfaces reported by stroud,[42] Lee and Seed,[45] Schofield and Wroth,[4] Atkinson and Bransby,[8] Sladen *et al.*,[21] and Coop and Lee[39] suggests that the following general stress–state relation may be used to describe the state boundary surface for a variety of soils:

$$\left(\frac{\eta}{M}\right)^n = 1 - \frac{\xi}{\xi_R} \tag{10}$$

where n is a new material constant which typically ranges between 1·0–5·0 and the reference state parameter is $\xi_R = (\lambda - \kappa)\ln r$.

For the sake of simplicity, the original and modified Cam-clay models use the same r value for all soil types although in reality this is not the case. In particular, recent experimental data suggests that the values of the spacing ratio for sand are generally much higher than those for clay.[39] In this paper, the spacing ratio r is allowed to vary with material types. As will be shown later, this assumption can go a long way to overcome some of the most important drawbacks of the standard Cam-clay models mentioned earlier in the paper.

It is interesting to note that the stress–state relation (7) of the original Cam-clay model can be recovered from equation (10) by choosing $n = 1$ and $r = 2{\cdot}7183$. Figure 4 shows the stress–state relations for three different n values. In addition, the 'wet' side of the stress–state relation (9) for modified Cam-clay can also be matched accurately with equation (10) by choosing $r = 2{\cdot}0$ in conjunction with a suitable n value (typically around 1·5–2·0).

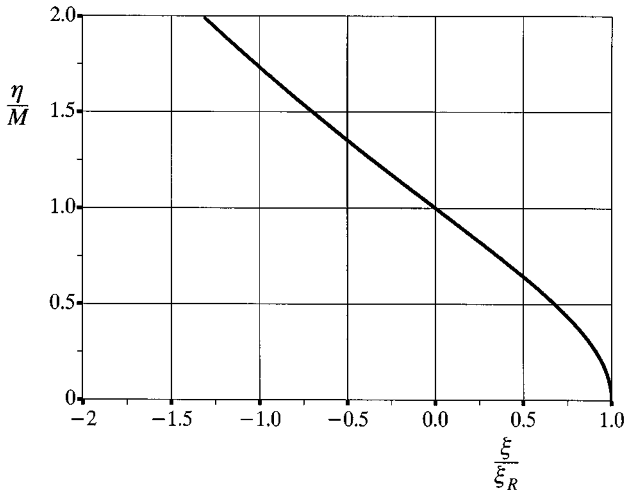

Figure 3. Stress-state relation for the modified Cam-clay model with $\xi_R = (\lambda - \kappa)\ln 2$

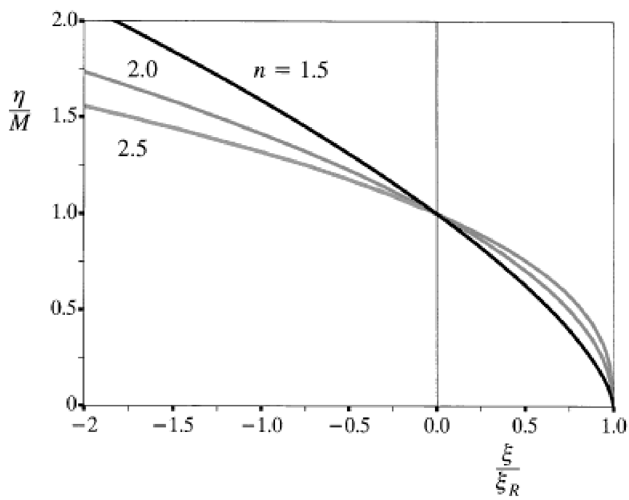

Figure 4. General stress–state relations with $\xi_R = (\lambda - \kappa)\ln r$

Based on Been's experimental correlations between the peak friction angle and the initial state parameter,[22] Collins,[46] Collins *et al.*[47] and Yu[48,49] have proposed various plasticity models for sand in which the angles of friction and dilation are assumed to be an exponential function of the state parameter. While these existing state parameter models are very successful in modelling dense sands, they may not be valid for very loose granular materials as the plastic deformation of the soil during consolidation cannot be realistically taken into account. Recently, Jefferies[27] has

used a linear relationship between the peak stress ratio (i.e. the peak friction angle) and the state parameter in developing sand models, which may be regarded as special cases of the general stress–state relation (10) proposed in this paper. While the simple rigid state parameter model, Nor-Sand, proposed by Jefferies[27] proves to be satisfactory for modelling sand behaviour under drained loading conditions, it may not be able to model the behaviour of sand under undrained loading conditions.

From Figure 1, it can be shown that

$$\frac{\xi}{\xi_{\mathrm{R}}} = \frac{-(\lambda-\kappa)\ln(p'_x/p')}{(\lambda-\kappa)\ln r} = 1 + \frac{\ln(p'/p'_0)}{\ln r} \tag{11}$$

Substituting the above equation into the general stress–state relation (10) leads to a generalized yield surface in terms of preconsolidation pressure p'_0 (i.e. the state boundary surface along the elastic wall) as follows:

$$\left(\frac{\eta}{M}\right)^n = -\frac{\ln(p'/p'_0)}{\ln r} \tag{12}$$

The state boundary surface (12) for $r = 4$ is plotted in Figure 5 for three different n values. It is observed that they look very similar to experimental yielding surfaces for sands reported by Nova and Wood,[28] Tatsuoka and Ishihara,[29] Sladen *et al.*,[21] and Coop and Lee.[39] Figure 5 is able to reproduce an important feature of the observed yield surface for sand and that is the deviatoric stress often reaches a local peak before approaching the critical state. This feature has been theoretically predicted by Chandler[50] using an energy-based plasticity theory which allowed for volume changes due to particle deformation and particle rearrangement.

Following similar procedures to those used above, the stress–state relation (10) can also be expressed as the state boundary surface normalized by equivalent critical state and consolidation

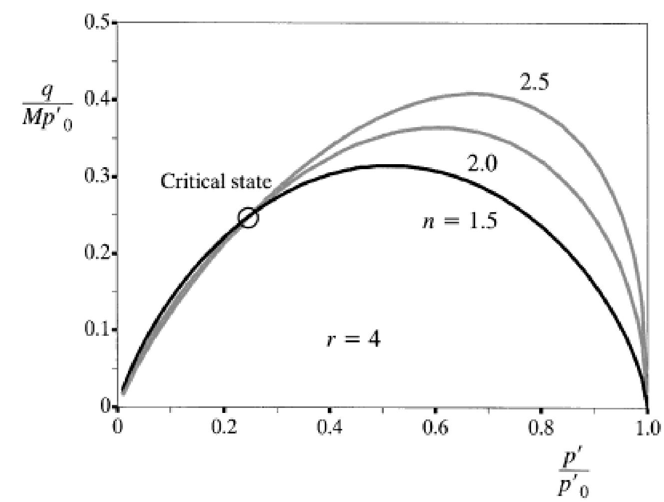

Figure 5. State boundary surfaces normalized by preconsolidation pressure

pressures p'_u, p'_e (i.e. the state boundary surfaces at constant v) :

$$\left(\frac{\eta}{M}\right)^n = 1 - \frac{\ln(p'/p'_u)}{\Lambda \ln r} \tag{13}$$

$$\left(\frac{\eta}{M}\right)^n = -\frac{\ln(p'/p'_e)}{\Lambda \ln r} \tag{14}$$

where $\Lambda = (\lambda - \kappa)/\lambda$ is known as the plastic volumetric strain ratio.[4] The state boundary surfaces (14) and (13) for $r = 4$ and $\Lambda = 0.9$ are plotted in Figures 6 and 7, respectively, for three different n values.

INCREMENTAL STRESS–STRAIN RELATIONS

The basic assumption of the state parameter model is the existence of a critical (or steady) state at which the soil deforms without any plastic volume change. The material behaviour prior to the achievement of the critical state is assumed to be controlled by the state parameter. The state parameter model proposed in this paper is an elastic–plastic strain hardening (or softening) model, that postulates that a soil specimen can be considered as an isotropic continuum. It is assumed that the strain rate tensor is the sum of an elastic, reversible component and of a plastic, irreversible part, that is:

$$\dot{\varepsilon}_{\mathrm{p}} = \dot{\varepsilon}_p^{\mathrm{e}} + \dot{\varepsilon}_p^{\mathrm{p}} \tag{15}$$

$$\dot{\varepsilon}_{\mathrm{q}} = \dot{\varepsilon}_q^{\mathrm{e}} + \dot{\varepsilon}_q^{\mathrm{p}} \tag{16}$$

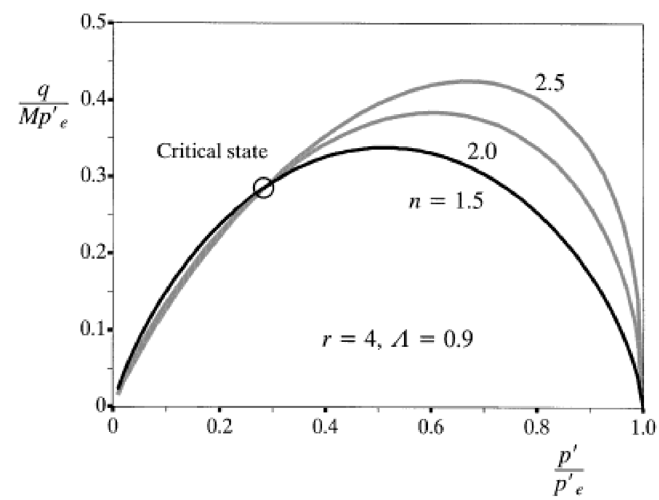

Figure 6. State boundary surfaces normalized by equivalent consolidation pressure

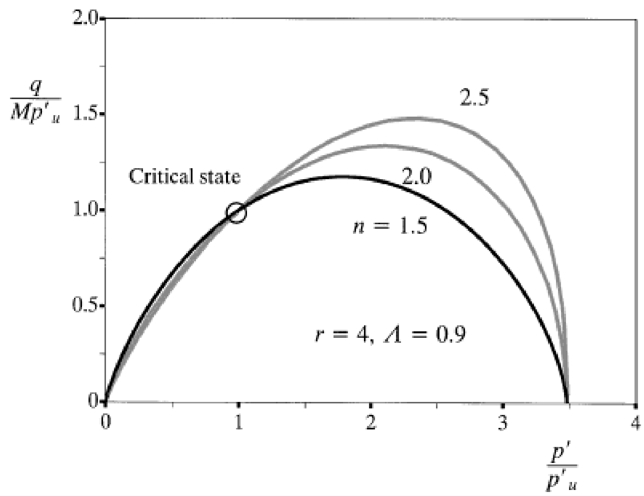

Figure 7. State boundary surfaces normalized by equivalent critical state pressure

It is noted that the void ratio v can be eliminated from equation (5) using the following expression:

$$\frac{\dot{v}}{v} = -\dot{\varepsilon}_p \tag{17}$$

where v is specific volume and ε_p denotes volumetric strain. The above equation can be integrated to express the state parameter as a function of the mean stress and plastic volumetric strain:

$$\xi = v_0 \exp(-\varepsilon_p^{\mathrm{e}} - \varepsilon_p^{\mathrm{p}}) + \lambda \ln p' - \Gamma \tag{18}$$

Suppose now that the plastic behaviour of the materials can be modelled by a yield function

$$f(p', q, \xi) = 0 \tag{19}$$

and a corresponding plastic potential is defined by

$$g(p', q, \beta) = 0 \tag{20}$$

where β is a parameter controlling the size of the plastic potential which passes through the current stress state.

Following the usual procedure, a complete elastic–plastic stress–strain relationship can be obtained as follows:

$$\dot{\varepsilon}_{\mathrm{p}} = \frac{\dot{p}}{K} + \frac{g_{,p}}{H} \times [(f_{,p} + f_{,\xi}\xi_{,p})\,\dot{p} + f_{,q}\,\dot{q}] \tag{21}$$

$$\dot{\varepsilon}_{\mathrm{q}} = \frac{\dot{q}}{3G} + \frac{g_{,q}}{H} \times [(f_{,p} + f_{,\xi}\,\xi_{,p})\,\dot{p} + f_{,q}\,\dot{q}] \tag{22}$$

where H is the plastic hardening modulus defined as

$$H = -f_{,\xi}\,\xi_{,\varepsilon_p^{\rm p}}\,g_{,p} \tag{23}$$

and $g_{,p} = \partial g/\partial p$ and $g_{,q} = \partial g/\partial q$ etc.; K and G are used to denote the bulk and shear moduli of the soils.

Like the standard Cam-clay models, the state parameter theory outlined above is a volumetric strain hardening plasticity model. While the use of both volumetric and deviatoric plastic strain hardening[51,52] may be able to capture the strong dilation during hardening prior to failure in dense sands, a few more material constants will have to be introduced into the model. Recently, Bardet[53] and Crouch *et al.*[54] demonstrated that volumetric plastic strain hardening models can also be used successfully to model sand behaviour in a wide stress region. Since the primary aim of this paper is to present a simple unified critical state theory for both clay and sand, the volumetric plastic strain hardening of the standard Cam-clay models has been preserved.

UNIFIED STATE PARAMETER MODEL FOR CLAY AND SAND

Based on the general incremental stress–strain relation presented in the previous section, this section describes a simple, unified constitutive model for both clay and sand. This simple model is referred to as CASM (standing for Clay And Sand Model).

Elastic component of the model

As in the standard Cam-clay models, the present state parameter model assumes that the bulk modulus is proportional to the mean effective pressure p':

$$K = \frac{vp'}{\kappa} \tag{24}$$

The second independent elastic constant is chosen by using either an assumed constant value of Poisson's ratio μ or an assumed constant value of shear modulus G. As it is usually more convenient to specify a value of Poisson's ratio, the shear modulus may therefore be assumed to vary with stress level in the same way as K

$$G = \frac{3(1-2\mu)}{2(1+\mu)}\,K \tag{25}$$

From a theoretical point of view, it would be preferable to assume a constant value of shear modulus, as Zytynski *et al.*[55] showed that the use of a constant Poisson's ratio would lead to a non-conservative model in the sense that it may not conserve energy during closed stress cycles. It should be noted that this effect may not be so important for static problems since the artificial energy dissipation caused by the constant Poisson's ratio model is very small compared with the energy dissipation by plastic strains. Furthermore, Gens and Potts[6] pointed out that a constant G does not agree well with experimental observations and may imply negative values of Poisson's ratio at low stresses, which is physically unreasonable.

Stress–State relation and yield function

The choice of the constitutive functions for plastic potential and yield function is an important step in constitutive modelling, since strain rates depend essentially on their derivatives with respect to stresses.

In this paper, the general stress–state relation (10) proposed in a previous section has been adopted to describe the behaviour of soil yielding. In terms of effective mean and deviatoric stresses, and state parameter, the yield function takes the following form:

$$f(p', q, \xi) = \left(\frac{q}{Mp'}\right)^n + \frac{\xi}{\xi_R} - 1 = 0 \tag{26}$$

It is noted that when $n = 1$ and $\xi_R = (\lambda - \kappa)$, the above yield function reduces to the original Cam-clay yield surface. In addition, the modified Cam-clay yield surface can also be matched by the above yield function by choosing a certain value of n (which depends on the plastic volumetric strain ratio of the soil) in conjunction with $\xi_R = (\lambda - \kappa)\ln 2$.

As discussed previously, the use of the above stress–state relation to describe yielding of sand is supported by observations of experimentally determined yield surface for clays and sands. The above stress–state relation indicates that it is the normalized state parameter (i.e. the state parameter divided by a reference state parameter) rather than the state parameter itself that controls the size of yield surface. As a result, the present model provides a rational framework for modelling the behaviour of sands with different mineralogy, angularity and particle size.

The need for some form of normalization of state variables in constitutive modelling was also recognized by Desai and co-workers in their development of the disturbed state concept.[56–58] They use the relative intact (RI) state and the fully adjusted (FA) state to define a disturbance function which is assumed to control the soil behaviour. The present paper follows more closely the critical state concept and uses the isotropic normal consolidation line (NCL) and the critical state line (CSL) as two reference states in defining the normalized state parameter. It is interesting to note that the disturbance function used by Desai and Toth[56] [see Figure 2 and equation (8b) of their paper] is similar, in concept, to the normalized state parameter used in equation (26). The difference is that the present paper considers void ratios of various states at the same mean effective stress while void ratios of different states at the same strain level were used in Reference 56.

Stress–dilatancy relation and plastic potential

To define a plastic potential, it is necessary to use a stress–dilatancy relation which defines the relationship between stress ratio and dilatancy rate. Perhaps the most successful stress–dilatancy relation, which may now be considered to be one of the milestones in soil mechanics, is due to Rowe[59, 60]

$$d = \frac{\dot{\varepsilon}_p^{\mathrm{p}}}{\dot{\varepsilon}_q^{\mathrm{p}}} = \frac{9(M - \eta)}{9 + 3M - 2M\eta} \tag{27}$$

where d is known as the dilatancy rate. Rowe's stress–dilatancy relation, originally developed from minimum energy considerations of particle sliding, has met with greatest success in describing the deformation of sands and other granular media. As Rowe's stress–dilatancy

relation is very similar to the flow rule of the original Cam-clay model, it may also be used to describe the experimental stress–dilatancy data for clays.[30,61]

Although much effort has been devoted in the past to developing an even better stress–dilatancy relation for soils, very little progress seems to have been made in this front. On the other hand, Rowe's stresspt–dilatancy relation (either in its original or modified forms), which provides satisfactory results for most practical problems, has been widely accepted by geotechnical community.[27,62,63] Excessive attention to the fine details of the stress–dilatancy relation (or flow rule) for a soil may not be warranted, since, as rightly pointed out by Wroth and Houlsby,[64] it is invariably much easier to predict the ratios between strains rather than their absolute magnitudes. It is for this reason that Rowe's stress–dilatancy relation will be used in this paper to derive a unified plastic potential for both clay and sand.

Following the usual procedure, Rowe's stress–dilatancy relation (27) may be integrated to give the following plastic potential:

$$g(p', q, \beta) = 3M \ln \frac{p'}{\beta} + (3 + 2M) \ln \left(\frac{2q}{p'} + 3 \right) - (3 - M) \ln \left(3 - \frac{q}{p'} \right) = 0 \tag{28}$$

where the size parameter β can be determined easily for any given stress state (p', q) by solving equation (28). Note that the plastic flow rule adopted in CASM is non-associated, as the plastic potential is not identical to the yield surface.

State parameter and hardening rule

As discussed earlier, the hardening law used in this model is basically of the isotropic volumetric plastic strain hardening type. In particular, the size of yield surface is controlled by the state parameter which depends on plastic volumetric strains. Using equation (18), the volumetric plastic strain hardening law is shown to be:

$$\xi_{,\varepsilon_p^p} = -v \tag{29}$$

The plastic hardening modulus defined by (23) can be derived as follows:

$$H = \frac{v}{\xi_{\mathbf{R}}} \left[\frac{3M}{p'} - \left(\frac{6 + 4M}{2q + 3p'} + \frac{3 - M}{3p' - q} \right) \eta \right] \tag{30}$$

MODEL CONSTANTS AND THEIR IDENTIFICATION

There are a total of 7 material constants required in CASM. In the following sections, the role of each of these seven constants and the possible methods for determining them are briefly discussed.

Elastic constants–κ and μ

The elastic behaviour is modelled by the slope of the swell line κ and Poisson's ratio μ. A typical value of κ for sands is 0·005 and its value is generally much larger for clays,

ranging between 0·01 and 0·06. Poisson's ratio μ is typically in the range of 0·15–0·35 for clay and sand.

Critical state constants–λ, Γ and M

The critical state line for a soil is fully defined by constants λ, Γ and M. Measurement of these critical state constants is straightforward for clay soil, but for sand it proves to be much more difficult and special care needs to be exercised in determining them using triaxial testing.[38] Typical values of λ for sands at a relatively low-pressure level (say less than 1000 kPa) is between 0·01–0·05 and for some soils its value may be larger in a high-pressure region. The λ value for clay is usually in the range of 0·1–0·2. Γ is typically between 1·8–4·0 for various soils. Triaxial tests (drained and undrained with pore pressure measurement) on isotropically consolidated samples can be used to obtain the frictional constant M. It is necessary to continue these tests to large strains to ensure that the samples are close to the critical state. M is normally between 0·8–1·0 for clays, and 1·1–1·4 for sands.

Spacing ratio (or reference state parameter)–r (or ξ_R)

The spacing ratio r is used, in one way or another, to define the shape of the yield surface by all critical state constitutive models as well as many bounding surface plasticity models. In this paper, the spacing ratio is used to estimate the reference state parameter which corresponds to the loosest state a soil is likely to reach in practice. For the sake of simplicity, the standard Cam-clay models assume a single constant spacing ratio r for all soil types. In the original and modified Cam-clay models, r is fixed at 2·718 and 2·0, respectively. Although reasonable for clays, this simplification is found to be less successful for sands. In CASM, the assumption of a fixed space ratio for all soil types is abandoned and r is allowed to vary from 1 to ∞. Experimental data indicates that for clays, r typically lies in the range of 1·5–3·0 and for sands the value of r is generally much larger.[39, 54]

For most applications, it is satisfactory to treat the NCL as the reference consolidation line, and therefore the measurement of r for clays does not impose any difficulties as the NCL can be easily located. In contrast, locating the NCL for sands seems to be more difficult as a test device able to supply very high pressure is required.[8] However, as noted by Crouch *et al.*,[54] existing experimental evidence indicates that many quartz-based sands seem to share essentially the same NCL. If NCL for a given sand cannot be measured, it is acceptable to choose a positive state parameter (typically ranging between 0·05–0·2) that is unlikely to be encountered in practice as the reference state parameter.

Stress–state coefficient–n

The stress–state coefficient n used in the general stress–state relation (10) is a new material constant introduced in this paper. As discussed before, the value of n is typically between 1·0–5·0. To determine n for a given soil, it is necessary to plot the stress paths of a few triaxial tests (both drained and undrained) on soils of different initial conditions in terms of stress ratio η against the state parameter ξ.

Using the general stress–state relation (10), experimental state boundary surfaces should be regarded as a straight line in the plot of $\ln(1-(\xi/\xi_R))$ missingagainst $\ln(\eta/M)$. The stress–state coefficient n is the slope of the state boundary surface in this particular log–log plot.

PREDICTION AND VALIDATION

This section describes an application of CASM to predict the measured behaviour of clay and sand under both drained and undrained loading conditions.

Influence of initial conditions on computed stress–strain behaviour

Before using the model, CASM, to predict stress–strain behaviour of individual stress–strain curves, it is instructive to investigate the influence of initial conditions on the computed stress–strain relations for both clay and sand under drained and undrained loading conditions.

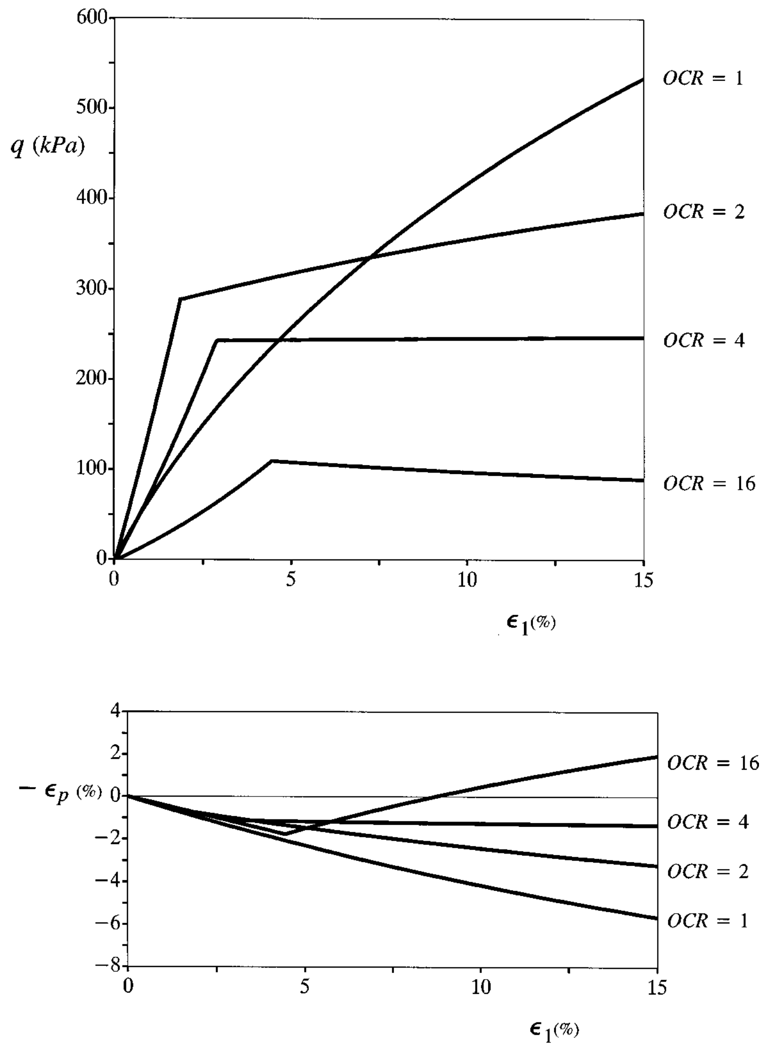

Figure 8. Effect of OCR on computed behaviour of drained triaxial compression tests on clay

Figures 8 and 9 show some aspects of the pattern of computed clay response during triaxial compression tests under drained and undrained conditions. The effects of different values of OCR (or initial state parameter) on the stress–strain relationship shown in the figures are generally in accordance with experimental observation. In this set of simulation, the material constants similar to those of London clay are used, and they are:

$$\lambda = 0{\cdot}161, \quad \Gamma = 2{\cdot}759, \quad \mu = 0{\cdot}3, \quad \kappa = 0{\cdot}062,$$

$$M = 0{\cdot}888, \quad r = 3{\cdot}0, \quad n = 2{\cdot}0$$

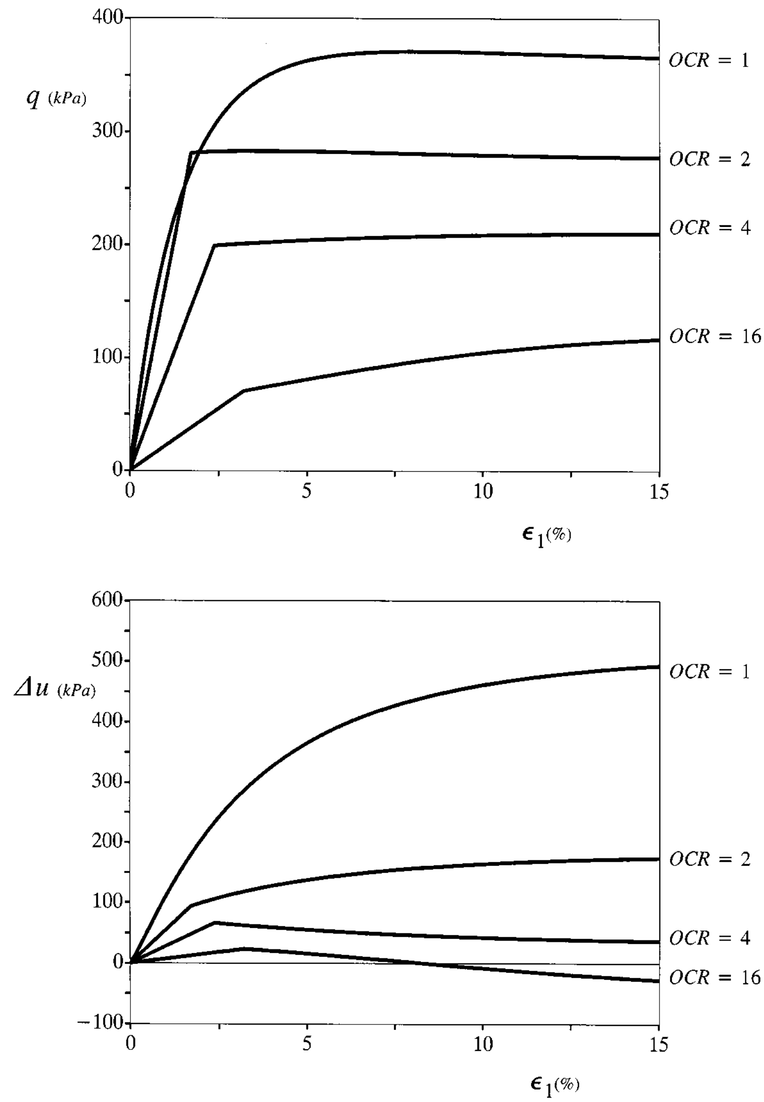

Figure 9. Effect of OCR on computed behaviour of undrained triaxial compression tests on clay

Plotted in Figures 10 and 11 are computed stress–strain response of sand during triaxial compression tests under drained and undrained conditions. The effects of different values of initial state parameter ξ_0 on the stress–strain relationship are shown. The material constants used are similar to those of Ticino sand which are given below:

$$\lambda = 0{\cdot}024, \quad \Gamma = 1{\cdot}986, \quad \mu = 0{\cdot}3, \quad \kappa = 0{\cdot}008,$$

$$M = 1{\cdot}29, \xi_R = 0{\cdot}075 \ (r = 108{\cdot}6), \quad n = 2{\cdot}0$$

To achieve better agreement with experimental data for undrained compression tests on sand at a state looser than critical, it is recommended that the initial state parameter of the sample be

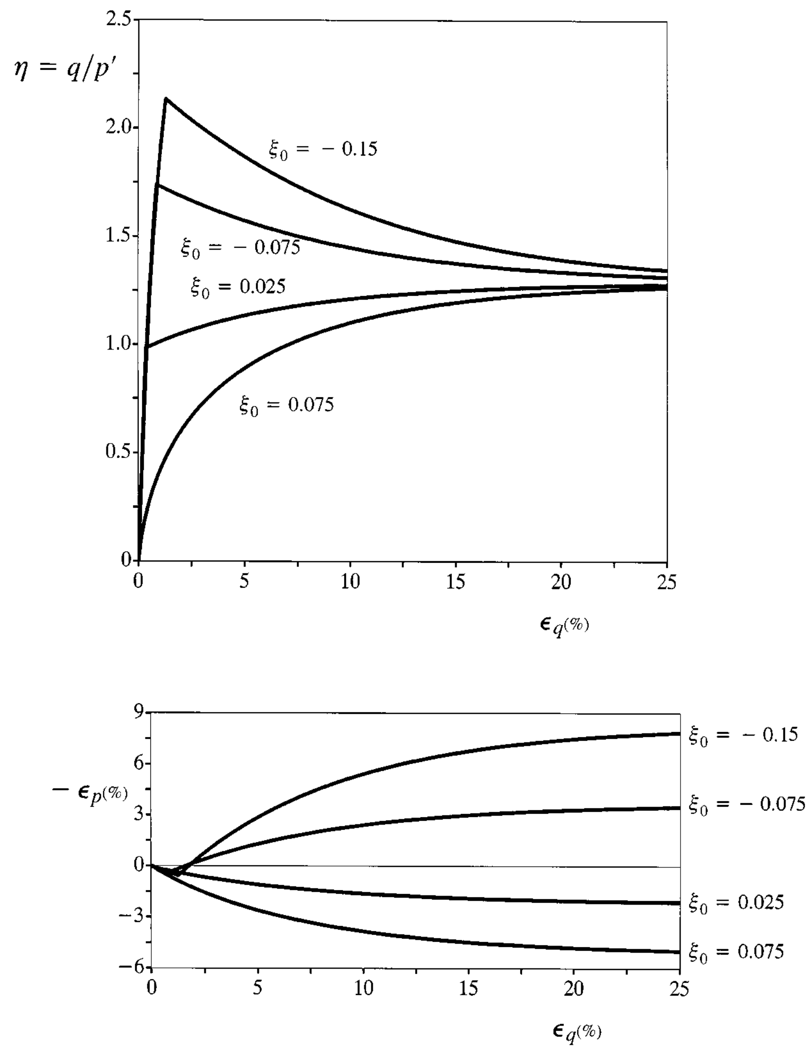

Figure 10. Effect of initial state parameter on computed behaviour of drained triaxial compression tests on sand

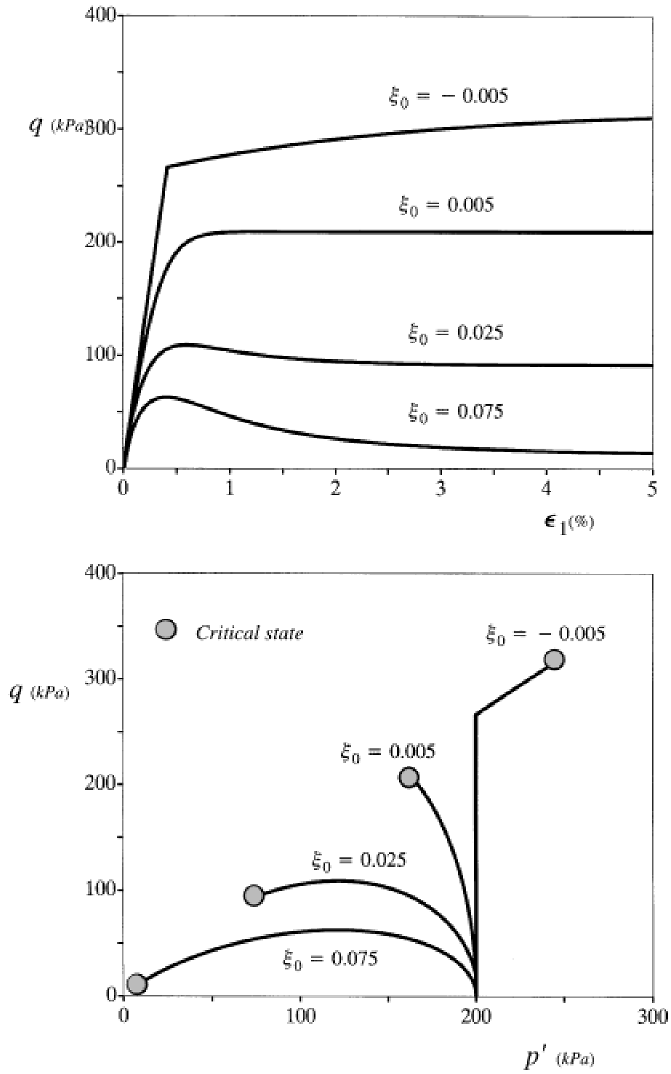

Figure 11. Effect of initial state parameter on computed behaviour of undrained triaxial compression tests on sand

taken as the reference state parameter (i.e. $\xi_R = \xi_0$) . The effect of model constant r and n on computed softening behaviour of sand is illustrated in Figure 12. It is clear that by varying the values of r and n, CASM can be satisfactorily used to model materials with different softening responses.

It can be seen from Figures 8–12 that CASM is capable of reproducing much of clay and sand-like stress–strain behaviour observed in the laboratory. Perhaps the only exception is for undrained tests on sand at a state denser than critical where it is often observed that the mean

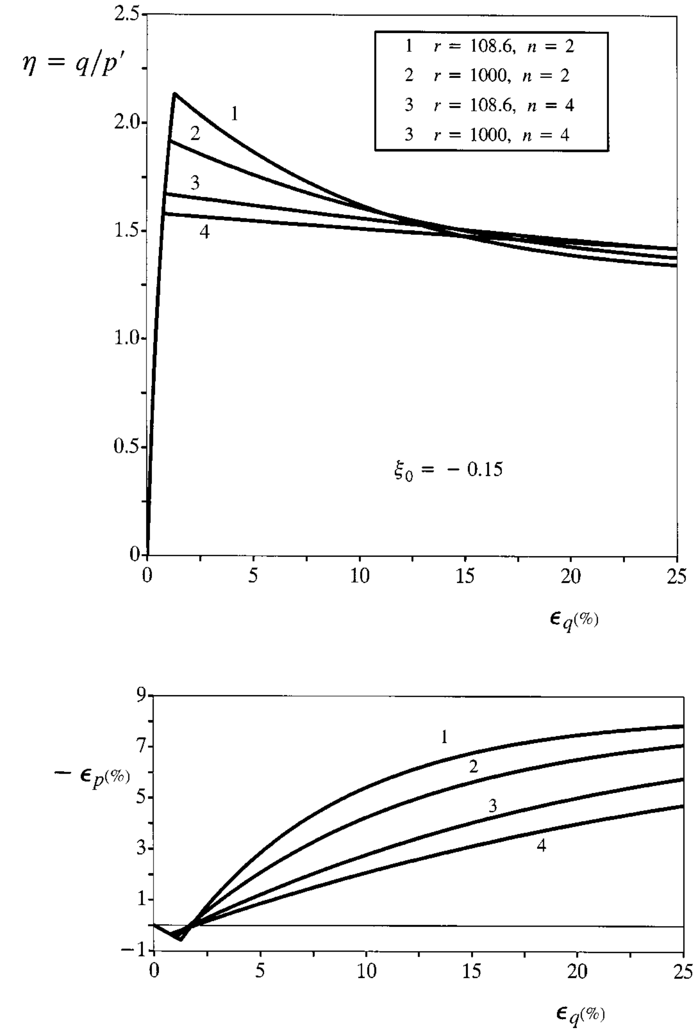

Figure 12. Effect of model constants r and n on computed softening response of drained triaxial compression tests on dense sand

effective stress tends to decrease a bit before increasing to the critical state. As shown in Figure 11, CASM does not predict such a pattern of change in the mean effective stress. This is a limitation associated with volumetric strain hardening where no plastic deformation is allowed within the state boundary surface. This difficulty can be easily overcome by introducing some additional plastic strains within the main yield surface in the model.[53,54] The penalty is that such a modification would certainly require a few more material constants.

To check how well CASM can predict individual stress–strain curves, some experimental data will now be used to compare with CASM.

Drained and undrained behaviour of normally and overconsolidated clays

To assess the performance of CASM for clay, test data from the classic series of tests performed on remoulded Weald clay at Imperial College, London[65] is used. Two of the four tests discussed are drained and two are undrained, while two of the tests are performed on normally consolidated samples with OCR of 1·0 and two on heavily overconsolidated samples with OCR of 24.

The material constants used in the prediction using CASM are as follows:

$$\lambda = 0{\cdot}093, \quad \Gamma = 2{\cdot}06, \quad \mu = 0{\cdot}3, \quad \kappa = 0{\cdot}025,$$

$$M = 0{\cdot}9, \xi_R = 0{\cdot}0679\ (r = 2{\cdot}714), \quad n = 4{\cdot}5$$

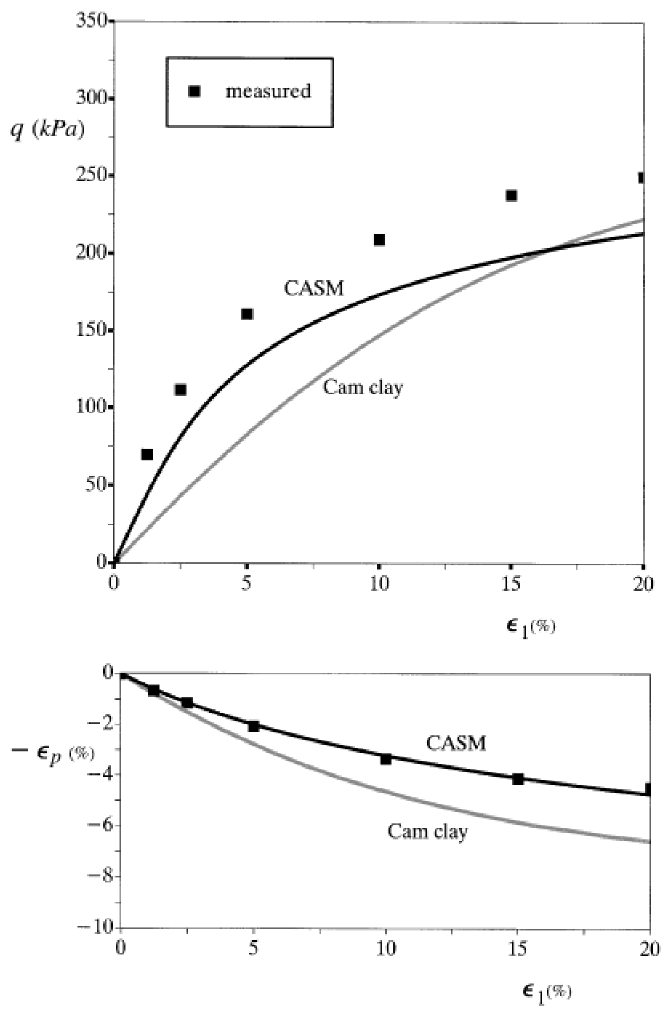

Figure 13. Model prediction for drained compression of a normally consolidated sample of Weald clay (OCR = 1, $v_0 = 1{\cdot}632$, $p'_0 = 207\,\text{kPa}$)

Note the NCL has been used as the reference consolidation line, and therefore the reference state parameter ξ_R is equal to the initial state parameter of the normally consolidated sample. The critical state constants for Weald clay are from Parry.[66]

Figures 13–16 present comparisons of the model predictions and the measured behaviour for both drained and undrained compression of normally and overconsolidated Weald clays. For comparison, the original Cam-clay model has also be used to predict the measured clay stress–strain behaviour. It is found that while Cam-clay is reasonable for modelling normally consolidated clays, it is not good for modelling overconsolidated clays. For the drained testing of the overconsolidated clay, Figure 14 suggests that Cam-clay gives a significant over-prediction of

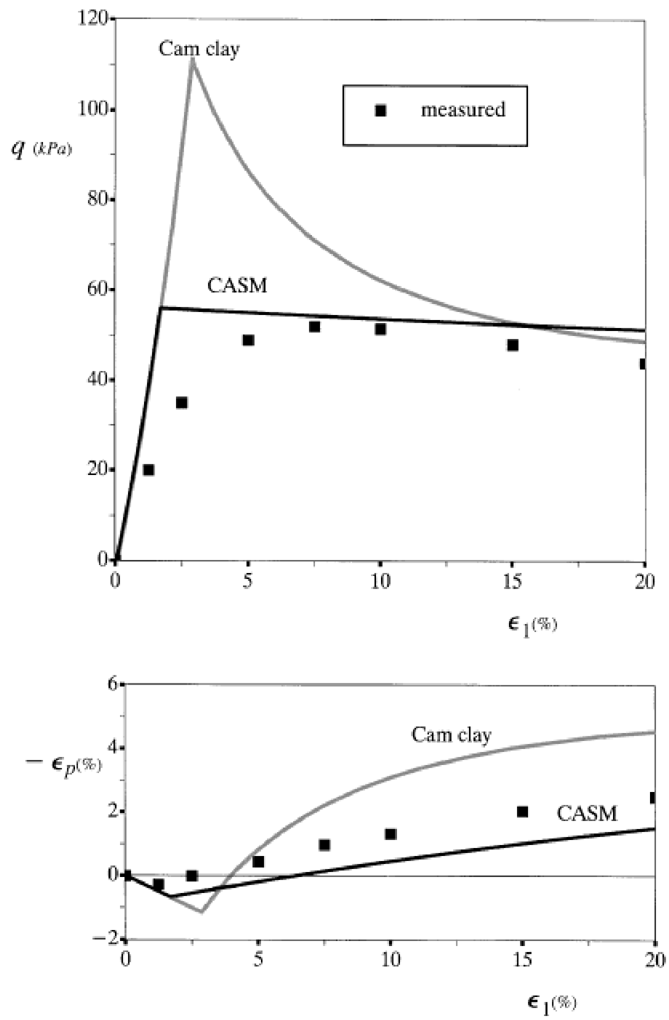

Figure 14. Model prediction for drained compression of a heavily overconsolidated sample of Weald clay (OCR = 24, $v_0 = 1{\cdot}617$, $p'_0 = 34{\cdot}5$ kPa)

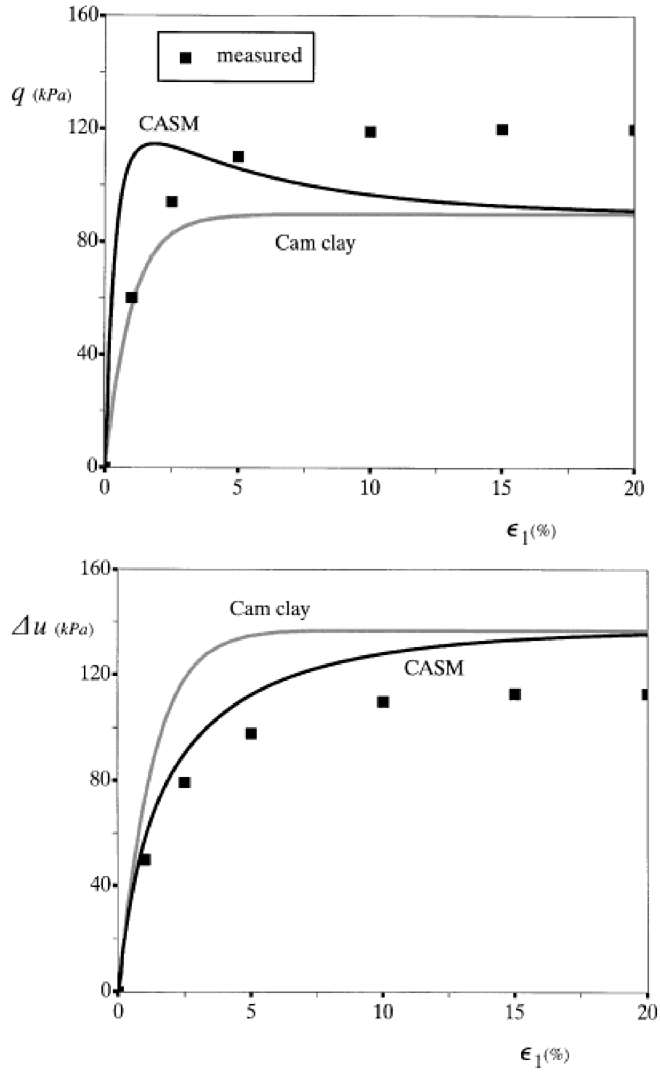

Figure 15. Model prediction for undrained compression of a normally consolidated sample of Weald clay (OCR = 1, $v_0 = 1{\cdot}632$, $p'_0 = 207\,\text{kPa}$)

the peak deviatoric stress and soil dilatancy. As for the undrained testing, it is evident from Figure 16 that Cam-clay under-predicts the shear strain at peak strength and over-predicts the negative excess pore pressure. In contrast, Figures 13–16 indicates that the predictions of CASM are consistently better than those by Cam clay for both normally and overconsolidated clays. In particular, CASM is found to be able to capture reasonably well the overall behaviour of the overconsolidated clay observed in the laboratory.

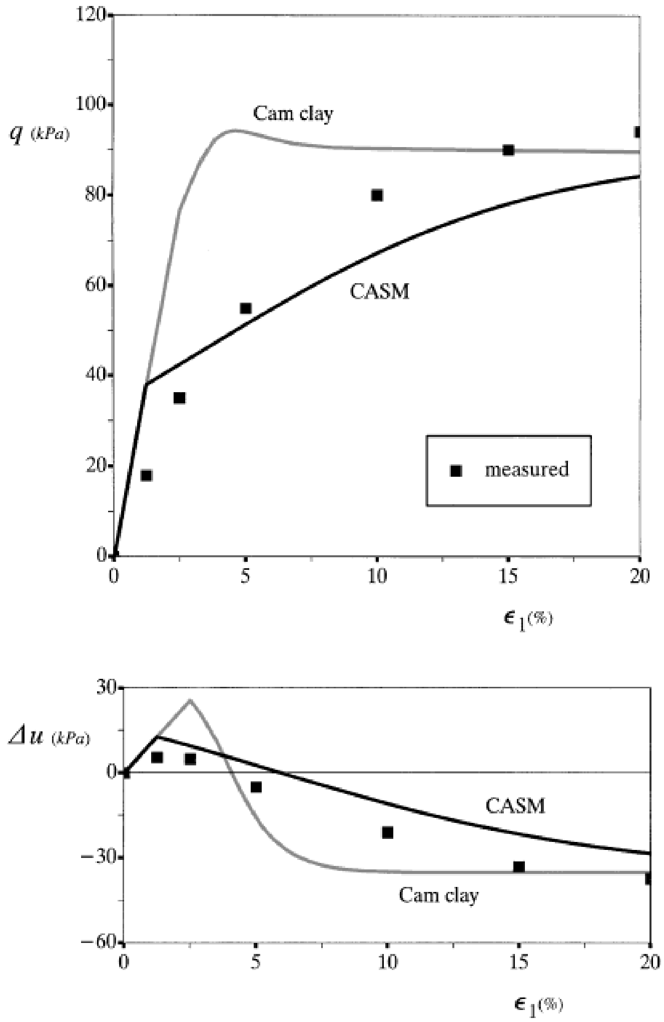

Figure 16. Model prediction for undrained compression of a heavily overconsolidated sample of Weald clay (OCR = 24, $v_0 = 1{\cdot}617$, $p'_0 = 34{\cdot}5$ kPa)

It should also be pointed out that for undrained testing of a normally consolidated clay, Figure 15 shows that CASM predicts a strain softening towards the critical state after a peak strength has been reached. Although this behaviour is not apparent in the results of the undrained test of the remoulded Weald clay, compelling evidence has however led Bishop[20] to conclude that the soil reach a peak strength before approaching to the critical state is a well-established behaviour for undrained tests on normally consolidated samples of many undisturbed cohesive soils. A similar behaviour was also observed by Allman and Atkinson[67] in their results of undrained testing of reconstituted Bothkennar soil. As will be discussed later, this is also a well-known result from the undrained testing of a vary loose sand.

Drained behaviour of loose, medium, and dense sands

To check the performance of CASM for sand, test data reported by Been *et al.*[38] and Jefferies[27] on a predominantly quartz sand with a trace of silt known as Erksak 330/0·7 will be used. Three tests are selected to compare with CASM. These tests are on the densest sample D667 (with an initial void ratio of 1·59 at the initial cell pressure of 130 kPa), the loosest sample D684 (with an initial void ratio of 1·82 at the initial cell pressure of 200 kPa) and a medium dense sample D662 (with an initial void ratio of 1·677 at the initial cell pressure of 60 kPa).

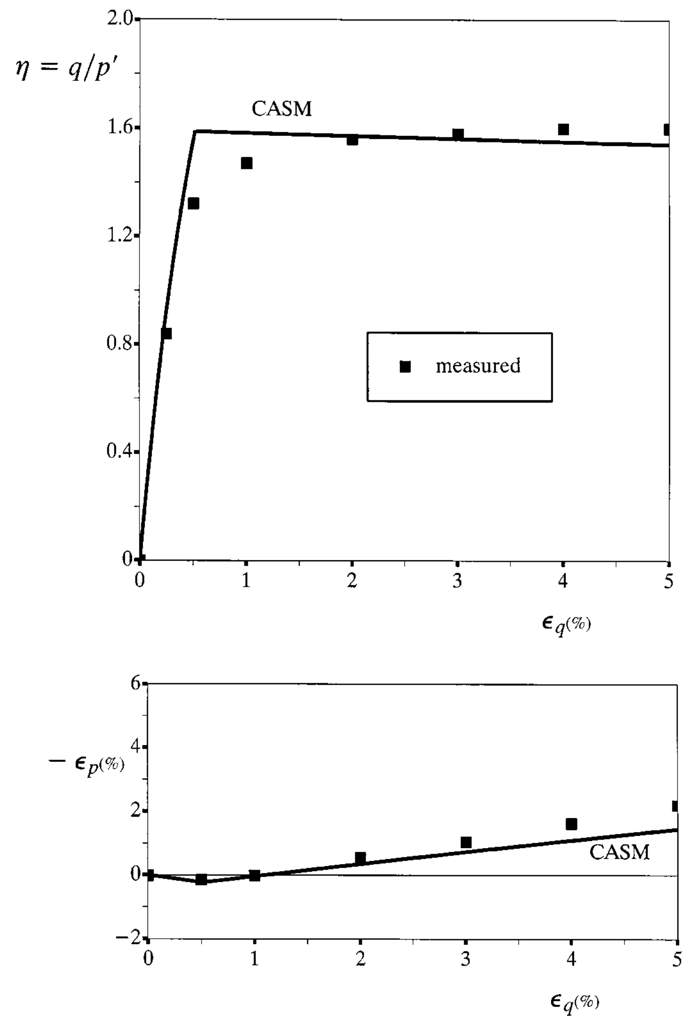

Figure 17. Model prediction for drained compression of a very dense sample of Erksak 330/0·7 sand (D667: $v_0 = 1{\cdot}59$, $p'_0 = 130\,\mathrm{kPa}$)

The material constants used in the CASM predictions are as follows:

$$\lambda = 0{\cdot}0135, \quad \Gamma = 1{\cdot}8167, \quad \mu = 0{\cdot}3, \quad \kappa = 0{\cdot}005, \quad M = 1{\cdot}2,$$

$$\xi_R = 0{\cdot}075 \ (r = 6792), \quad n = 4{\cdot}0$$

In order to allow for the prediction for sands from the loosest to the densest states, the reference state parameter ξ_R for CASM is assumed to be equal to the initial state parameter of the loosest sample D684. The critical state constants for Erkask sand are from Been *et al.*[38] and Jefferies.[27] As the accurate elastic constants are not known for Erksak sand, some typical values are used in the prediction.

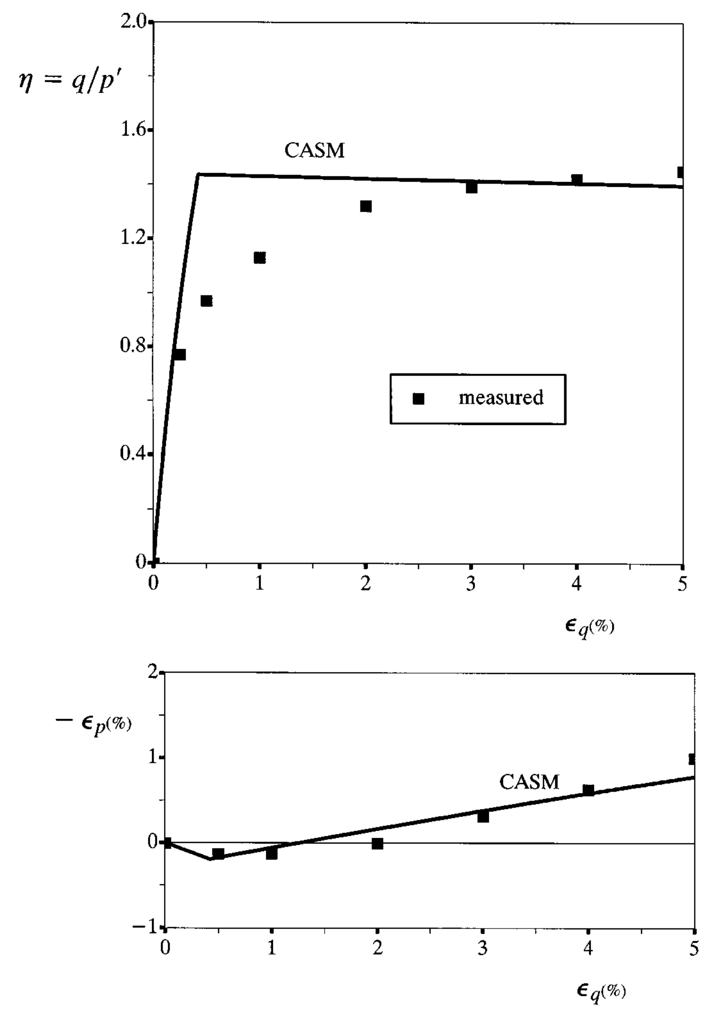

Figure 18. Model prediction for drained compression of a medium sample of Erksak 330/0·7 sand (D662: $v_0 = 1{\cdot}677$, $p'_0 = 60\,\mathrm{kPa}$)

Figures 17–19 present comparisons of the predictions and the measured behaviour for tests on the samples D667, D662 and D684, respectively. It is clear from these figures that overall CASM is quite satisfactory for predicting measured behaviour on loose, medium and dense sands. It is noted that one obvious deficiency with CASM is that it tends to under-predict the shear strain at peak strength. This is due to the fact that, like Cam clay, CASM does not allow any plastic deformation to develop within the state boundary surface.

Undrained behaviour of very loose sand

The term 'very loose' is used here to describe sand in a state which is much looser than its critical state. It is well known that very loose sands can collapse and strain soften during

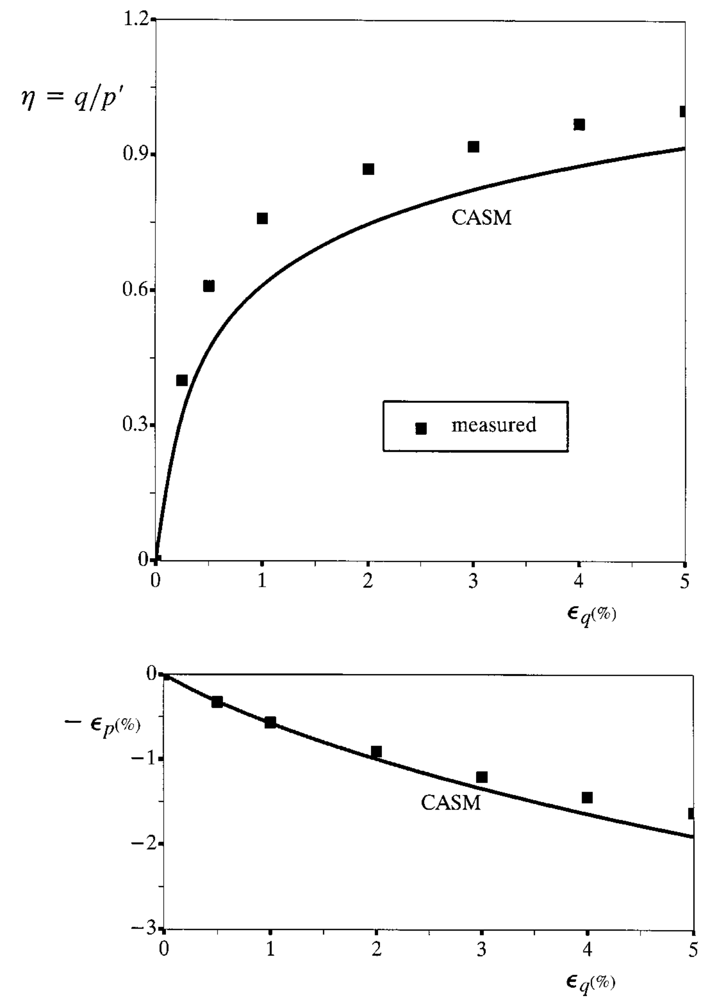

Figure 19. Model prediction for drained compression of a very loose sample of Erksak 330/0·7 sand (D684: $v_0 = 1{\cdot}82$, $p'_0 = 200\,\mathrm{kPa}$)

monotonic undrained loading and ultimately reach a critical (or steady) state. During monotonic undrained loading loose sand reaches a peak resistance and then rapidly strain softens to a steady state, and this is a condition necessary for liquefaction to occur. As discussed in the introduction, most existing critical state models, such as Cam-clay and Nor-Sand, are unable to model this behaviour.

To demonstrate the applicability of CASM for modelling undrained behaviour of a very loose sand, test data obtained by Sasitharan *et al.*[68] on Ottawa sand is used. Four tests have been selected to compare with CASM. These tests are on the samples with initial void ratios of 0·793 and 0·804. Different initial mean effective stresses were used in these tests.

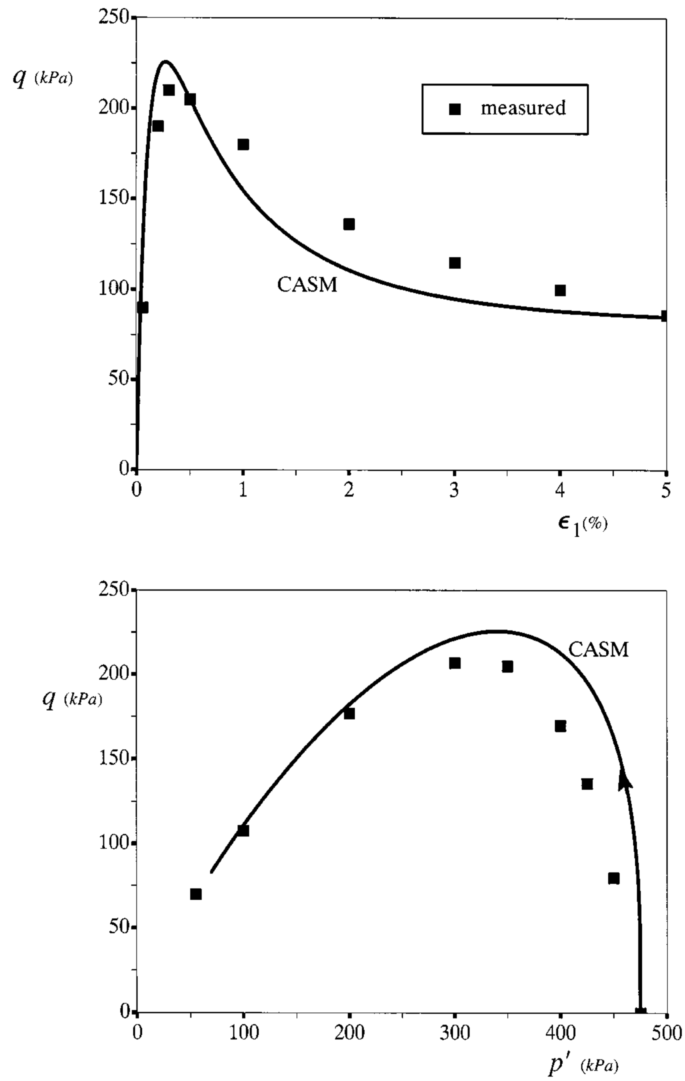

Figure 20. Model prediction for undrained compression of a very loose Ottawa sand ($v_0 = 1{\cdot}793$, $p'_0 = 475\,\mathrm{kPa}$)

The material constants used in the CASM predictions are as follows:

$$\lambda = 0{\cdot}0168, \quad \Gamma = 1{\cdot}864, \quad \mu = 0{\cdot}3, \quad \kappa = 0{\cdot}005, \quad M = 1{\cdot}19, \quad \xi_R = \xi_0, \quad n = 3{\cdot}0$$

The critical state constants for Ottawa sand are from Sasitharan *et al.*[76] Again the accurate elastic constants are not known for this sand and some typical values have to be used in the prediction. When CASM is used to model the undrained behaviour of a very loose sand, the reference state parameter ξ_R can be assumed to be equal to the initial state parameter of each sample. As will be

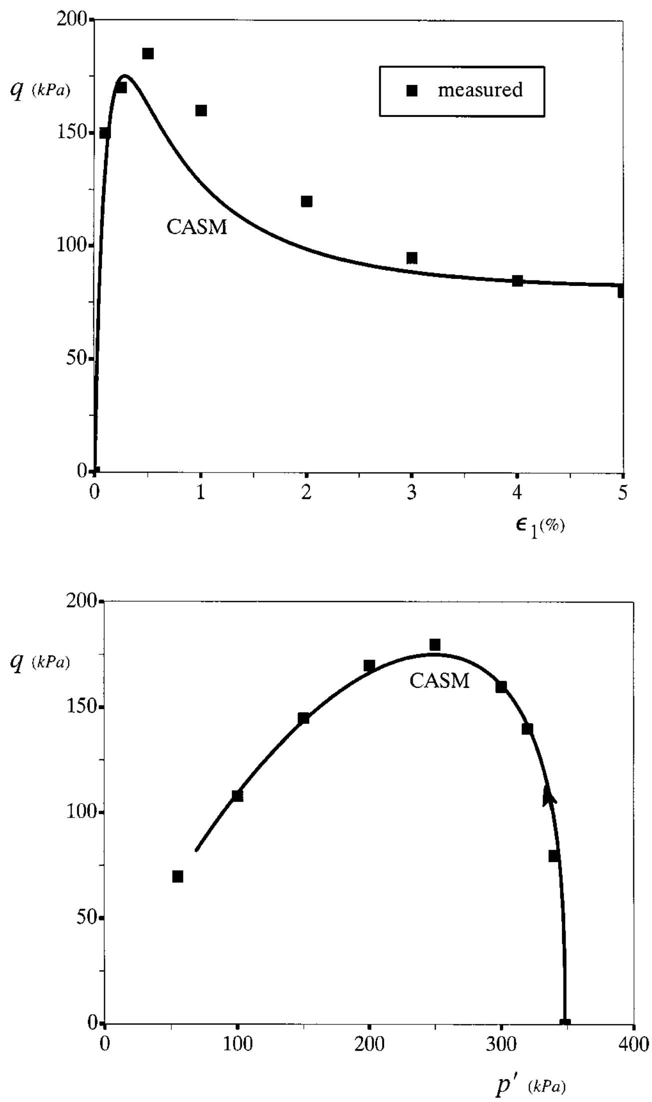

Figure 21. Model prediction for undrained compression of a very loose Ottawa sand ($v_0 = 1{\cdot}793$, $p'_0 = 348\,\text{kPa}$)

shown below, this assumption proves to be very satisfactory for predicting undrained behaviour of very loose sands.

Presented in Figures 20–23 are comparisons of the CASM predictions and the measured behaviour for undrained tests on the four very loose samples. It is evident from these figures that CASM can be satisfactorily used to predict the measured behaviour of undrained tests on very loose sands. In particular, CASM predicts that the peak strength is developed at a very small axial strain and afterwards the response shows a marked strain softening with increase in axial strain before approaching the critical state.

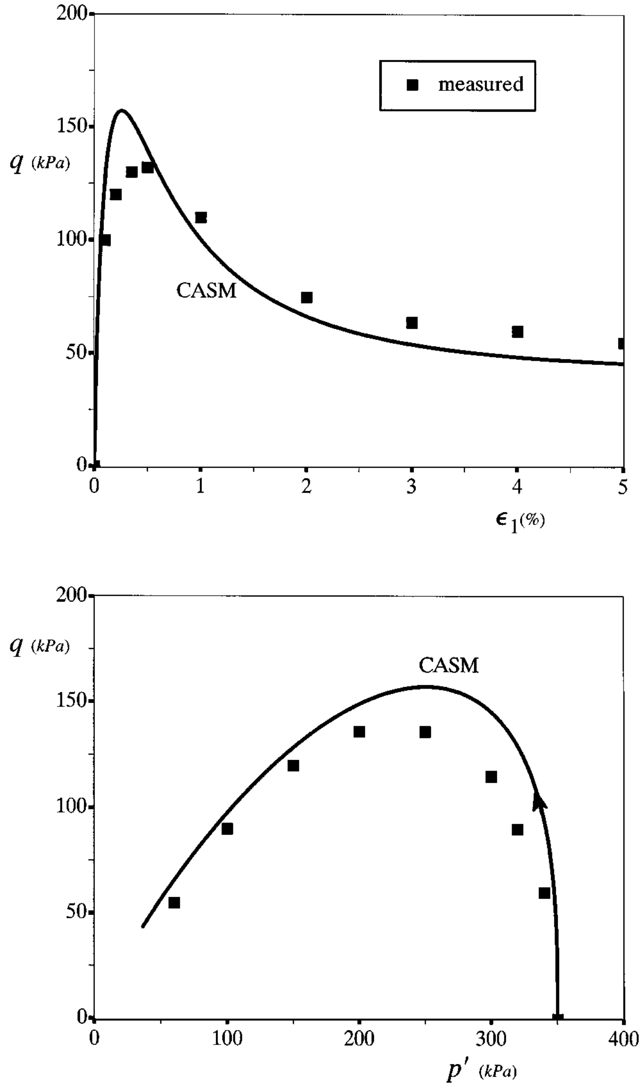

Figure 22. Model prediction for undrained compression of a very loose Ottawa sand ($v_0 = 1{\cdot}804$, $p'_0 = 350\,\mathrm{kPa}$)

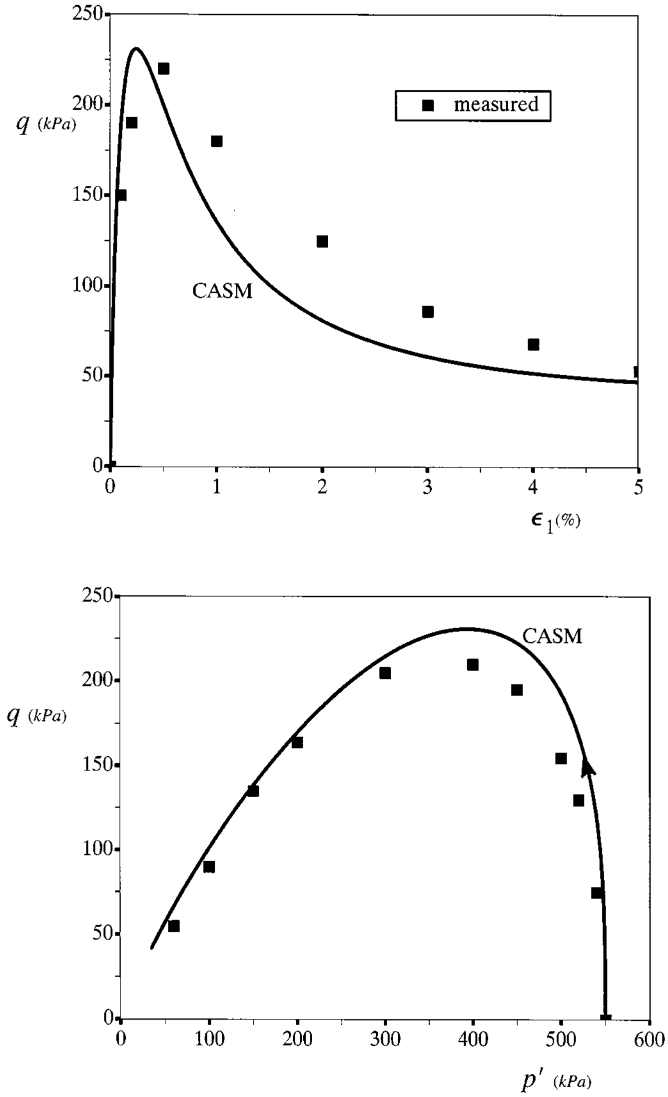

Figure 23. Model prediction for undrained compression of a very loose Ottawa sand ($v_0 = 1{\cdot}805$, $p'_0 = 550\,\mathrm{kPa}$)

CONCLUDING REMARKS

The aim of this paper is to present a simple and unified formulation of the constitutive equations for both clay and sand. From a practical point of view, this is particularly advantageous since when applied in the numerical analysis of a boundary value problem, only different material constants need to be incorporated either for clay or sand, and the form of the constitutive equations remains the same. The present model, CASM, has been developed by using a general

stress ratio-state parameter relation as the yield surface. Rowe's stress–dilatancy relation is used as the plastic potential and the resulting plastic flow rule is therefore non-associated. While the original Cam-clay yield surface is a special case of the CASM yield locus, the modified Cam clay yield surface can also be approximated by CASM. Comparisons with experimental data suggest that CASM is able to capture the overall behaviour of clay and sand observed under both drained and undrained loading conditions, and therefore represents a very useful extension of Cam clay that is known to be only valid for normally consolidated clays.

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